# APPROXIMATE EQUATIONS OF CREEP OF THIN SHELLS

## (PRIBLIZHENNYE URAVNENIIA POLZUCHESTI TONKOSTENNYKH OBOLOCHEK)

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In view of the known analogy [1] between the equations of stationary creep and the equations of the theory of elastic-plastic deformations (or nonlinear elasticity), the expressions for the forces and moments in terms of the deformations of the middle surface - derived for an elastic-plastic shell on the basis of Kirchhoff's hypothesis [2] - are applicable for the analysis of creep in shells. In the solution of specific problems, certain simplifications are introduced [3,4] by various arguments, sometimes not quite consistently [5]. The theorem of nesting surfaces of constant rate of energy dissipation, established recently by Drucker and Calladine [6,7], facilitates an approach to this problem from a more general point of view. As a result, it is shown that it is possible to derive simplified relations between the forces and moments and the deformations, whose form allows for an effective utilization of the variational methods, in particular, the method given by Kachanov [8].

**1.** We shall assume the power law of creep, which results in the equations of stationary creep in the form [1]

$$\dot{\varepsilon}_x = B_1 \sigma_x^{\ m} \qquad \text{(uniaxial tension)} \tag{1.1}$$

$$\dot{\varepsilon}_{ij} = \frac{\partial \Lambda}{\partial \sigma_{ij}} \qquad \text{(general case)} \tag{1.2}$$

where

$$\Lambda = 3^{\frac{m+1}{2}} \frac{B_1}{m+1} T^{m+1}, \qquad T = \frac{1}{\sqrt{2}} \sqrt{\sigma_{ij}\sigma_{ij} - 3\sigma^2}, \qquad \sigma = \frac{1}{3}\sigma_{mn}\delta_{mn}$$

The rate of dissipation of energy per unit volume, $D$, is given by the expression

$$D = \sigma_{ij}\dot{\varepsilon}_{ij} = \sigma_{ij}\frac{\partial\Lambda}{\partial\sigma_{ij}} = (m+1)\,\Lambda \tag{1.3}$$

Let the deforming body be subjected to the loading by concentrated forces or moments $Q_i$ acting at certain points.

We introduce, instead of $B_1$, two new constants $\sigma_N$ and $\dot{\varepsilon}_N$ connected by the relation

$$\dot{\varepsilon}_N = B_1\sigma_N{}^m \tag{1.4}$$

and, comparing the average rate of dissipation of energy in the volume $V$ and the "nominal" rate of dissipation $D_N = \sigma_N\dot{\varepsilon}_N$, we obtain

$$\frac{1}{V}\int D\,dV = \sigma_N\,\dot{\varepsilon}_N \tag{1.5}$$

The equation (1.5) determines a hypersurface in the rectangular coordinates $Q_i$ and $\sigma_N$.

The theorem of Drucker and Calladine states that a surface (1.5) constructed for a given value of $m$ is enclosed by another surface corresponding to a smaller value of $m$.

**2.** We shall consider first the case of a cylindrical shell with axially symmetrical loading, which allows for a simple geometrical interpretation. The rates of deformation in the axial and circumferential directions are given by the known expressions

$$\dot{\varepsilon}_x = \dot{\varepsilon}_1 + \dot{\varkappa}_1 z, \qquad \dot{\varepsilon}_\varphi = \dot{\varepsilon}_2 \qquad \left(\dot{\varkappa}_1 = -\frac{d^2 w}{dx^2}\right) \tag{2.1}$$

where $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ are the rates of extension of the middle surface and $\dot{\kappa}_1$ is the rate of curvature of the generatrix.

Using (2.1), we calculate the rate of dissipation of energy $D = D^*$ in the volume corresponding to the unit area of the middle surface

$$D^* = \int_{-h/2}^{h/2} (\sigma_x\dot{\varepsilon}_x + \sigma_\varphi\dot{\varepsilon}_\varphi)\,dz = T_1\dot{\varepsilon}_1 + T_2\dot{\varepsilon}_2 + M_1\dot{\varkappa}_1. \tag{2.2}$$

Here, $T_1$ and $T_2$ are the axial and circumferential forces, $M_1$ is the bending moment. We shall assume that the axial force vanishes, i.e. $T_1 = 0$. For an elastic material

$$M_1 = \frac{Eh^3}{12(1-\nu^2)}\,\dot{\varkappa}_1, \qquad T_2 = Eh\dot{\varepsilon}_2$$

Substituting these relations into (2.2) and assuming $\nu = 1/2$,

$E = \sigma_N/\dot{\varepsilon}_N$, we obtain the equation of the surface (1.5) for the case $m = 1$

$$t_2^2 + \frac{9}{16} m_1^2 = 1 \tag{2.3}$$

where

$$t_2 = \frac{T_2}{T_N}, \qquad m_2 = \frac{M_2}{M_N}, \qquad T_N = \sigma_N h, \qquad M_N = \frac{\sigma_N h^2}{4} \tag{2.4}$$

In the case $m \to \infty$, the surface (1.5) represents the corresponding yield condition [6]. For a cylindrical shell, with axially symmetrical loading (for $T_1 = 0$), this condition can be approximately represented in the form [2]

$$t_2^2 + \frac{3}{4} m_1^2 = 1 \tag{2.5}$$

Figure 1 shows the corresponding curve (continuous line), which only slightly differs from the exact condition [2] (broken line).

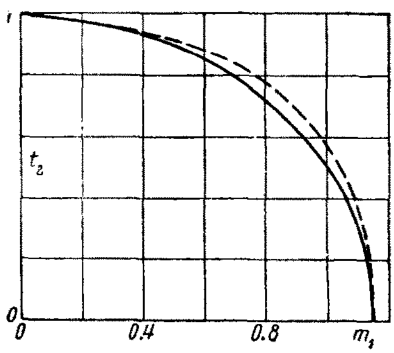


Fig. 1.

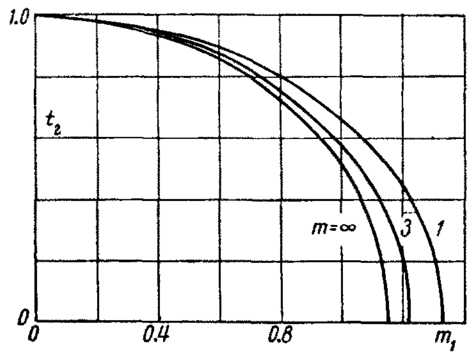


Fig. 2.

Figure 2 shows the curves (2.3) and (2.5). According to the theorem of Drucker and Calladine, similar curves for intermediate values of $m$ should be contained between these two. The anticipated location of the intermediate curves can be determined with considerable accuracy because the distance between the bounding curves is small, and in the region of largest divergence it is possible to find the points of intersection of all the curves with the axis $m_1$ using the known exact solutions for pure bending. We shall assume the intermediate curves in the form

$$f^2 \equiv t_2^2 + \alpha m_1^2 = 1 \tag{2.6}$$

where $\alpha$ is a constant. Applying to the considered shell element the generalized theorem of Castigliano [1], we obtain

$$\dot{\varkappa}_1 = \frac{\partial \Lambda^*}{\partial M_1}, \qquad \dot{\varepsilon}_2 = \frac{\partial \Lambda^*}{\partial T_2} \qquad \left(\Lambda^* = \frac{1}{m+1} D^*\right) \tag{2.7}$$

The vector with the components $\dot{\kappa}_1$, $\dot{\varepsilon}_2$ has the direction normal to the surface (2.6) at the corresponding point in the plane $M_1$, $T_2$ and, consequently

$$\dot{\varkappa}_1 : \dot{\varepsilon}_2 = \frac{\partial f}{\partial M_1} : \frac{\partial f}{\partial T_2} \tag{2.8}$$

The relations (2.7) and (2.8) are compatible if $\Lambda^* = \Lambda^*(f)$ is assumed.

From the comparison, for example, with the case of plane strain it follows that the function $\Lambda^*(f)$ should have the form

$$\Lambda^* = \frac{A}{m+1} f^{m+1} \tag{2.9}$$

Substituting the relations (2.7) into (2.2) and taking into account (2.9), we obtain the equation of the energy-dissipation surfaces in the form

$$Af^{m+1} = h\dot{e}_N \sigma_N \tag{2.10}$$

According to Fig. 2, the curves for an arbitrary $m$ should pass through the point $t_2 = 1$ for $m_1 = 0$. Since $f = 1$ for $m_1 = 0$ and $t_2 = 1$, we obtain from (2.10)

$$A = h\sigma_N \dot{e}_N \tag{2.11}$$

In order to determine the constant $\alpha$ we use the exact solution of the problem of pure bending (with $M_2 = 1/2M_1$), which has the form

$$M_1 = 3^{-\frac{1+\mu}{2}} \frac{h^{\mu+2}}{(2+\mu)} \frac{\sigma_N}{\dot{\varepsilon}_N^{\ \mu}} \dot{\varkappa}_1^{\ \mu} \qquad \left(\mu = \frac{1}{m}\right)$$

Calculating the corresponding rate of dissipation

$$D^* = M_1 \dot{\varkappa}_1 = 3^{\frac{m+1}{2}} \frac{(2+\mu)^m}{h^{1+2m}} \frac{\dot{\varepsilon}_N}{\sigma_N^{\ m}} M_1^{\ m+1}$$

and assuming it equal to the constant $h\sigma_N \dot{\varepsilon}_N$, we obtain

$$m_1 = \frac{4}{\sqrt{3}} (2+\mu)^{-\frac{1}{1+\mu}} \tag{2.12}$$

The curves (2.6) should intersect the axis $m_1$ at the points (2.12). From this condition we find

$$\alpha = \frac{3}{16}(2+\mu)^{\frac{2}{1+\mu}} \tag{2.13}$$

With the above value of $\alpha$ the energy-dissipation curves (2.6) coincide, for $m = 1$ and $m \to \infty$, with the curves (2.3) and (2.5), respectively. Figure 2 shows also one of the intermediate curves (for $m = 3$) constructed according to equation (2.6). In terms of physical variables in equations (2.7), and taking into account (2.6) and (2.9), we obtain

$$\begin{aligned} \dot{\varepsilon}_2 &= \frac{B_1}{h^m}\left[T_2^2 + \frac{16\alpha}{h^2}\ M_1^2\right]^{\frac{m-1}{2}} T_2 \\ \dot{\varkappa}_1 &= \frac{16\alpha B_1}{h^{2+m}}\left[T_2^2 + \frac{16\alpha}{h^2}\ M_1^2\right]^{\frac{m-1}{2}} M_1 \end{aligned} \tag{2.14}$$

3. The results obtained in the simple problem discussed above are helpful in the extension of the solution to a general case of a shell, which can be analyzed in similar manner. Calculating the rate of dissipation $D^*$ per unit area of the middle surface with the use of the usual kinematical hypotheses

$$\dot{\varepsilon}_{11} = \dot{\varepsilon}_1 + z\dot{\varkappa}_1 \qquad \dot{\varepsilon}_{22} = \dot{\varepsilon}_2 + z\dot{\varkappa}_2, \qquad \dot{\gamma}_{12} = \dot{\varepsilon}_{12} + z\dot{\varkappa}_{12} \tag{3.1}$$

and neglecting the ratio $h/R$ in comparison to one, we obtain

$$D^* = T_1\dot{\varepsilon}_1 + T_2\dot{\varepsilon}_2 + T_{12}\dot{\varepsilon}_{12} + M_1\dot{\varkappa}_1 + M_2\dot{\varkappa}_2 + M_{12}\dot{\varkappa}_{12} \tag{3.2}$$

where

$$\begin{aligned} T_1 &= \int_{-h/2}^{h/2} \sigma_1 dz, \qquad T_2 = \int_{-h/2}^{h/2} \sigma_2 dz, \qquad T_{12} = \int_{-h/2}^{h/2} \sigma_{12} dz \\ M_1 &= \int_{-h/2}^{h/2} \sigma_1 z dz, \qquad M_2 = \int_{-h/2}^{h/2} \sigma_2 z dz, \qquad M_{12} = \int_{-h/2}^{h/2} \sigma_{12} z dz \end{aligned} \tag{3.3}$$

The case $m = 1$ is equivalent to the problem of elasticity (with $\nu = 1/2$). Eliminating from (3.2) the rates of deformation of the middle surface by means of Love's relations, substituting the resulting expression into (1.5), and assuming $E = \sigma_N/\dot{\varepsilon}_N$, we obtain the equation of the energy-dissipation surface (for $m = 1$) in the form

$$(t_1^2 - t_1 t_2 + t_2^2 + 3t_{12}^2) + \frac{3}{4}(m_1^2 - m_1 m_2 + m_2^2 + 3m_{12}^2) = 1 \tag{3.4}$$

where the dimensionless forces and moments are defined by the relations (2.4).

In the case $m \to \infty$ the equation of the energy-dissipation surface coincides with the corresponding yield condition. Using an approximate yield condition [9]

$$(t_1^2 - t_1 t_2 + t_2^2 + 3t_{12}^2) + (m_1^2 - m_1 m_2 + m_2^2 + 3m_{12}^2) = 1 \tag{3.5}$$

We postulate the form of the intermediate surfaces as

$$f^2 \equiv (t_1^2 - t_1 t_2 + t_2^2 + 3t_{12}^2) + \frac{k}{16}(m_1^2 - m_1 m_2 + m_2^2 + 3m_{12}^2) = 1 \tag{3.6}$$

and, accordingly, we assume

$$\begin{aligned} \dot\varepsilon_1 &= \frac{\partial \Lambda^*}{\partial T_1}, & \dot\varepsilon_2 &= \frac{\partial \Lambda^*}{\partial T_2}, & \varepsilon_{12} &= \frac{\partial \Lambda^*}{\partial T_{12}} \\ \dot\varkappa_1 &= \frac{\partial \Lambda^*}{\partial M_1}, & \dot\varkappa_2 &= \frac{\partial \Lambda^*}{\partial M_2}, & \dot\varkappa_{12} &= \frac{\partial \Lambda^*}{\partial M_{12}} \end{aligned} \qquad \left(\Lambda^* = \frac{A}{m+1} f^{m+1}\right) \tag{3.7}$$

Determining the rate of dissipation $D^*$ with the formula (3.2), taking into account the relations (3.7), we obtain from the condition (1.5) the equation of the energy-dissipation surfaces

$$Af^{m+1} = h\dot\varepsilon_N \sigma_N \tag{3.8}$$

Here, $A$ has obviously the previous meaning, while the constant $k$ is determined by considering the case of pure bending (or comparing with the case of a cylindrical shell considered above)

$$k = 4(2+\mu)^{\frac{2}{1+\mu}} \tag{3.9}$$

Finally, the force-deformation relations (3.7) can be written as

$$\begin{aligned} \dot\varepsilon_1 &= \frac{B_1}{h^m} S^{m-1}\left(T_1 - \frac{1}{2}T_2\right), & \dot\varkappa_1 &= \frac{kB_1}{h^{m+2}} S^{m-1}\left(M_1 - \frac{1}{2}M_2\right) \\ \dot\varepsilon_2 &= \frac{B_1}{h^m} S^{m-1}\left(T_2 - \frac{1}{2}T_1\right), & \dot\varkappa_2 &= \frac{kB_1}{h^{m+2}} S^{m-1}\left(M_2 - \frac{1}{2}M_1\right) \\ \dot\varepsilon_{12} &= \frac{3B_1}{h^m} S^{m-1} T_{12}, & \dot\varkappa_{12} &= \frac{3kB_1}{h^{m+2}} S^{m-1} M_{12} \end{aligned} \tag{3.10}$$

where

$$S = \left[(T_1^2 - T_1T_2 + T_2^2 + 3T_{12}^2) + \frac{k}{h^2}(M_1^2 - M_1M_2 + M_2^2 + 3M_{12}^2)\right]^{1/2} \tag{3.11}$$

We also have

$$\Lambda^* = \frac{B_1 S^{m+1}}{(m+1)h^m} \tag{3.12}$$

Constructing the quadratic form

$$E = \left[\left(\dot\varepsilon_1^2 + \dot\varepsilon_1\dot\varepsilon_2 + \dot\varepsilon_2^2 + \frac{1}{4}\dot\varepsilon_{12}^2\right) + \frac{h^2}{k}\left(\dot\varkappa_1^2 + \dot\varkappa_1\dot\varkappa_2 + \dot\varkappa_2^2 + \frac{1}{4}\dot\varkappa_{12}^2\right)\right]^{1/2} \tag{3.13}$$

with (3.10) we obtain

$$E = \frac{\sqrt{3}}{2}\frac{B_1}{h^m} S^m, \qquad S = \left(\frac{2}{\sqrt{3}}\right)^{\mu} \frac{h}{B_1{}^{\mu}} E^{\mu} \tag{3.14}$$

With these relations, equations (3.10) can be solved with respect to the forces and moments and given in the form

$$\begin{aligned} T_1 &= \vartheta h \frac{E^{\mu-1}}{B_1{}^{\mu}} (2\dot{\varepsilon}_1 + \dot{\varepsilon}_2), & M_1 &= \frac{\vartheta h^3}{k} \frac{E^{\mu-1}}{B_1{}^{\mu}} (2\dot{\varkappa}_1 + \dot{\varkappa}_2) \\ T_2 &= \vartheta h \frac{E^{\mu-1}}{B_1{}^{\mu}} (2\dot{\varepsilon}_2 + \dot{\varepsilon}_1), & M_2 &= \frac{\vartheta h^3}{k} \frac{E^{\mu-1}}{B_1{}^{\mu}} (2\dot{\varkappa}_2 + \dot{\varkappa}_1) \qquad \left(\vartheta = 2^{\mu} 3^{-\frac{1+\mu}{2}}\right) \\ T_{12} &= \frac{\vartheta h}{2} \frac{E^{\mu-1}}{B_1{}^{\mu}} \dot{\varepsilon}_{12}, & M_{12} &= \frac{\vartheta h^3}{2k} \frac{E^{\mu-1}}{B_1{}^{\mu}} \dot{\varkappa}_{12} \end{aligned} \tag{3.15}$$

Introducing the dissipation function

$$L^* = \frac{A_1}{\mu + 1} E^{\mu+1}, \qquad A_1 = \frac{2\vartheta h}{B_1{}^{\mu}} \tag{3.16}$$

we can write equations (3.15) also in the following form

$$\begin{aligned} T_1 &= \frac{\partial L^*}{\partial \dot{\varepsilon}_1}, & T_2 &= \frac{\partial L^*}{\partial \dot{\varepsilon}_2}, & T_{12} &= \frac{\partial L^*}{\partial \dot{\varepsilon}_{12}} \\ M_1 &= \frac{\partial L^*}{\partial \dot{\varkappa}_1}, & M_2 &= \frac{\partial L^*}{\partial \dot{\varkappa}_2}, & M_{12} &= \frac{\partial L^*}{\partial \dot{\varkappa}_{12}} \end{aligned} \tag{3.17}$$

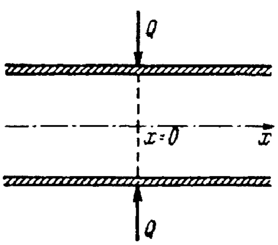


Fig. 3.

Having equations (3.7) and (3.17) we derive, in the usual way, the variational principles

$$\begin{aligned} \delta \left\{ \iint L^* dF - A_e \right\} &= 0 \\ \delta \left\{ A_e - \iint \Lambda^* dF \right\} &= 0 \end{aligned} \tag{3.18}$$

where $A_e$ is the power of external (edge and surface) loadings. In the first of equations (3.18), the variations are meant for kinematically admissible fields of velocities of the middle surface, while in the second, for statically admissible fields of forces and moments. Using Hill's method, it is possible to show that, for the actual state, the first functional (3.18) has an absolute minimum, and the second functional has an absolute maximum.

**4.** We shall use the first variational principle (3.18) for the problem of creep of a cylindrical shell with circumferential line loading (Fig. 3). Since the axial force is missing, the first of equations (3.15) gives

$$2\dot{\varepsilon}_1 + \dot{\varepsilon}_2 = 0 \tag{4.1}$$

In a cylindrical shell of radius $a$, with axially symmetrical loading

$$\dot{\varepsilon}_2 = \frac{w}{a}, \quad \dot{\varkappa}_1 = -\frac{d^2w}{dx^2}, \quad \dot{\varkappa}_2 = 0 \tag{4.2}$$

where $w$ is the radial velocity of creep. With (4.1) and (4.2), we obtain from the formula (3.13)

$$E = \left[\frac{3}{4}\dot{\varepsilon}_2^2 + \frac{h^2}{k}\dot{\varkappa}_1^2\right]^{1/2} \tag{4.3}$$

for which

$$L^* = \frac{hB_1^{-\mu}}{1+\mu}\left(\dot{\varepsilon}_2^2 + \frac{4}{3}\frac{h^2}{k}\dot{\varkappa}_1^2\right)^{\frac{1+\mu}{2}} \tag{4.4}$$

We shall assume $w$ in the form of the elastic solution [11]

$$w = w_0 e^{-\alpha x}(\cos\alpha x + \sin\alpha x) \tag{4.5}$$

The first variational principle (3.18) becomes

$$\delta\left\{w_0^{\mu+1}\int_{-\infty}^{\infty}\Phi_1(\alpha)\,dx - Qw_0\right\} = 0 \qquad \left(\Phi_1(\alpha) = L^*\left(\frac{w}{w_0}\right)\right) \tag{4.6}$$

Varying the parameters $\alpha$ and $w_0$, we obtain from (4.6) the two equations

$$Q = D(\alpha)\,w_0^{\mu}, \qquad \frac{d}{d\alpha}D(\alpha) = 0 \qquad \left(D(\alpha) = 2(1+\mu)\int_0^{\infty}\Phi_1(\alpha)\,dx\right) \tag{4.7}$$

Here, $D(\alpha)$ has the meaning of the "rigidity" of the shell.

Substituting (4.5) into (4.4), and taking into account (4.6) and (4.7), we obtain the expression

$$D(\alpha) = \frac{2hB_1^{-\mu}}{a^{1+\mu}}\int_0^{\infty}\left[(\cos\alpha x + \sin\alpha x)^2 + \frac{4}{3k}(2\alpha^2 ah)^2(\cos\alpha x - \sin\alpha x)^2\right]^{\frac{\mu+1}{2}} e^{-(1+\mu)\alpha x}\,dx \tag{4.8}$$

This can be represented in the following form

$$D = \frac{hB_1^{-\mu}}{\alpha_* a^{1+\mu}}V_m(\beta) \qquad \left(\alpha_* = \frac{1}{2}\frac{(3k)^{1/4}}{\sqrt{ah}}, \quad \beta = \frac{\alpha}{\alpha_*}\right) \tag{4.9}$$

$$V_m(\beta) = \frac{1}{\beta}\int_0^{\infty}[(1+\sin\zeta) + \beta^4(1-\sin\zeta)]^{\frac{1+\mu}{2}}\exp\left(\frac{-(1+\mu)\zeta}{2}\right)d\zeta \tag{4.10}$$

The diagram of the function $V_m(\beta)$ can be constructed (for a fixed $m$) by evaluating the integral on the right-hand side of (4.10) by numerical methods. The minimum of this curve corresponds to the root $\beta = \beta_0$. The values of $\beta_0$ found in this way and the corresponding values $\alpha_0 = \alpha_* \beta_0$ are shown in Fig. 4. The same figure contains also the diagram of the function $V_m^{\circ} = V_m(\beta)$ which determines, according to (4.9), the sought rigidity of the shell.

We note that, according to the diagram in Fig. 4, the values of $\beta_0$ do not differ significantly from one; it is exactly $\beta_0 = 1$ for $m = 1$. The minimum of the function $V_m(\beta)$ is most important. Assuming $\beta = \beta_0 \approx 1$, the integral (4.10) can be easily calculated, resulting in

$$V_m(1) = V_m^* = \frac{1}{1+\mu} 2^{\frac{3+\mu}{2}} \qquad (4.11)$$

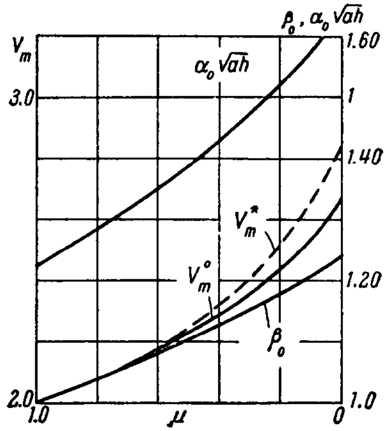


Fig. 4.

The curve $V_m^*$, shown in Fig. 4, differs from $V_m^{\circ}$, in the most unfavorable case $\mu = 0$, only by about 6 per cent.

Substituting $V_m^*$, instead of $V_m^{\circ}$, into the formula (4.9), we obtain a simple approximate expression for the rigidity $D$:

$$D = \frac{hB_1^{-\mu}}{\alpha_* a^{1+\mu}} \frac{1}{1+\mu} 2^{\frac{3+\mu}{2}} \qquad (4.12)$$

For $\mu = 1$, $B_1^{-1} = E$, the expression (4.12) reduces to the known exact solution for an elastic shell [11].

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